

1. Given that $z_1 = a + ib$, where a and b are real, and that $z_2 = 3 - i$,

(a) find $\frac{z_1}{z_2}$ in the form $x + iy$, expressing the real numbers x and y in terms of a and b . **(3)**

Given that $b = -2a$ and that $a > 0$,

(b) show that $|z_1| = a\sqrt{5}$, **(2)**

(c) find the value of $\arg \frac{z_1}{z_2}$. **(3)**



Question 1 continued

Lined area for writing the answer to Question 1.

(Total 8 marks)

Q1



2. $f(x) = x \cos x - 2x + 5$

(a) Show that $f(x) = 0$ has a root α in the interval $[2, 2.1]$. (2)

(b) Taking 2 as a first approximation to α , apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α , giving your answer to 2 decimal places. (5)

(c) Show that your answer to part (b) gives α **correct** to 2 decimal places. (2)



3. Given that $5 + 2i$ is a complex root of the equation

$$x^3 - 12x^2 + cx + d = 0, \quad c, d \in \mathbb{R},$$

- (a) write down the other complex root of the equation. (1)

- (b) Find the value of c and the value of d . (5)

- (c) Show the three roots of this equation on an Argand diagram. (2)



Question 3 continued

20 horizontal lines for writing.

Q3

(Total 8 marks)



4. Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 5 \cos t .$$

(10)



5. (a) Express $\frac{1}{(2r+1)(2r+5)}$ in partial fractions.

(2)

(b) Hence show, using the method of differences, that

$$\sum_{r=1}^n \frac{1}{(2r+1)(2r+5)} = \frac{n(8n+c)}{15(2n+3)(2n+5)},$$

where c is a constant to be found.

(6)



6.

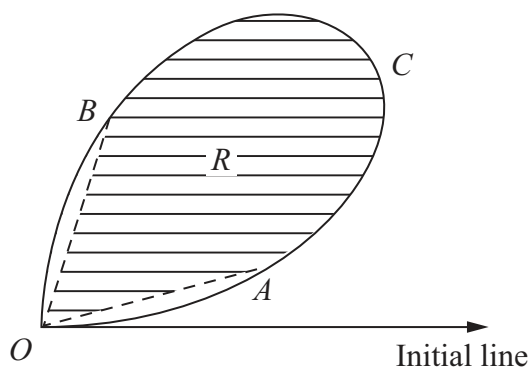


Figure 1

Figure 1 shows a sketch of the curve C with polar equation

$$r = a \sin 2\theta, \quad 0 \leq \theta \leq \frac{\pi}{2},$$

where a is a positive constant.

At the points A and B on C , $r = \frac{1}{2}a$.

- (a) Find the polar coordinates of A and B . (3)

The shaded region R , shown in Figure 1, is bounded by OA , OB and the arc AB of C .

- (b) Use integration to find the area of R , giving your answer in terms of a and π . (6)



7.

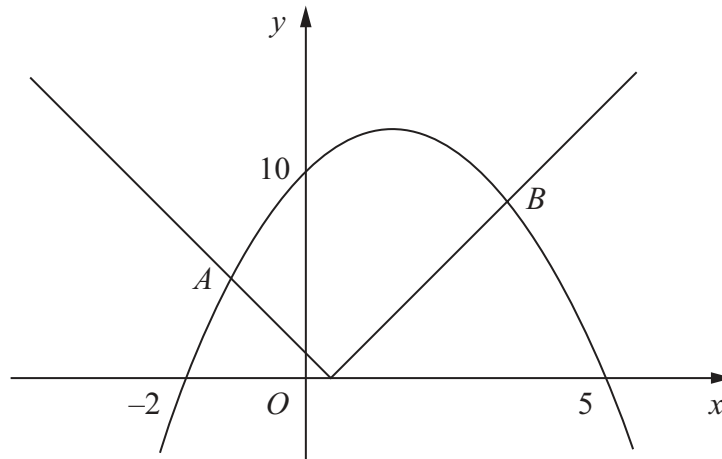


Figure 2

Figure 2 shows the graph of $y = 10 + 3x - x^2$ and the graph of $y = |3x - 1|$.

The graphs intersect at the points A and B .

(a) Use algebra to find the exact x -coordinates of A and B . (5)

(b) Find the set of values of x for which

$$10 + 3x - x^2 > |3x - 1|.$$
(2)

(c) Find the set of values of x for which $|10 + 3x - x^2| < |3x - 1|$. (3)



